A Reminder of the Limitations in Using Beer’s Law to Estimate Daily Radiation Interception by Vegetation

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ABSTRACT

Radiation extinction coefficients for leaf canopies are often calculated using Beer’s Law based on midday measurements of radiation interception. However, the assumption of Beer’s Law is not appropriate for leaf canopies and this empirical approach needs to be used with caution. Alternatively, classic derivations of radiation interception by leaf canopies have resulted in a similar exponential form defined as a function of time of day, day of year, and latitude. A common experimental approach of determining the extinction coefficient from midday measurements results in a minimum coefficient that underpredicts the total daily radiation interception. Two approaches are explored to improve the estimates of daily radiation interception under a clear sky. The first approach compared extinction coefficients calculated for midday against ones calculated as representative of radiation interception over the entire day. A linear correlation was found between the two extinction coefficients so that the midday extinction coefficient could be corrected to obtain a coefficient representative of the entire day under a clear sky. The second approach estimated the time of day when measurements could be made to obtain directly an extinction coefficient representative of the entire day. While such times in the day could be identified, this approach is impractical because the periods for taking these measurements are short and dependent on leaf angle.

The amount of solar radiation intercepted by vegetation has long been recognized as a key variable in estimating C assimilation rate and growth of plants. In recent years, growth of vegetation over fixed time intervals is often expressed directly as a function of intercepted radiation through the calculation of radiation use efficiency, which is the ratio of canopy growth divided by intercepted solar radiation (Sinclair and Muchow, 1999). Consequently, there is renewed interest in determining the daily amount of solar radiation intercepted by various leaf canopies.

Direct measurement of the daily intercepted radiation is a challenge because instruments need to be placed under leaf canopies and left there for the entire period for which radiation use efficiency is to be determined. Such an experiment involving several treatments and several replications would require permanent installation of a number of radiation sensors requiring a large investment in equipment and in time required to maintain sensors. One solution frequently used to overcome the equipment limitation is to use only one or a few sensors to make spot measurements under leaf canopies at one time of day, usually midday, as representative of radiation interception for the entire day. Such radiation interception data are often interpreted by deriving a radiation extinction coefficient in an exponential radiation-interception equation analogous to the Beer–Bouguer–Lambert Law.

\[ R_b/R_o = 1 - \exp(-kL), \]

where:

- \( R_b \) = irradiance transmitted through the leaf canopy,
- \( R_o \) = irradiance incident on the leaf canopy,
- \( k \) = radiation extinction coefficient, and
- \( L \) = leaf area index.

Monsi and Saeki (1953) originally suggested that the exponential equation as a function of the leaf area in horizontal layers in leaf canopies was a useful expression of vegetation radiation interception.

A key assumption in Beer’s Law, however, is that radiation passes through a medium containing small absorbing and scattering particles distributed uniformly throughout the medium. The basic assumption of Eq. [1], however, is not met in natural vegetation because the canopy is composed of discrete, contiguous leaf surfaces with characteristic angles of display. Further, the angle of the incident direct beam radiation for leaf canopies changes through the day as solar elevation changes. Measurements made near midday, for example, are taken at maximum solar elevation resulting in the greatest penetration of radiation through the leaf canopy. The bias of the midday measurements toward maximum radiation penetration, and hence maximum radiation use efficiency, is dependent on a number of factors, including latitude, time of year, and time of day (Goudriaan, 1982), and on the angles of the leaf segments. Muchow (1985) pointed out that small values for fraction of intercepted solar radiation measured at midday resulted in the greatest deviations from the midday value at other times of the day. Charles-Edwards and Lawn (1984) presented a correction factor to apply to midday measures to obtain a daily integral of light interception but the correction was developed empirically. Their experimental results were obtained for crops sown in rows and, hence, the results should not necessarily be expected to be stable across differing row widths or among species with differing canopy structures.

Abbreviations: G, leaf shadow projection; \( k_i \), instantaneous extinction coefficient; \( k_b \), daily extinction coefficient; \( L \), leaf area index; PAR, photosynthetically active radiation; \( R_b \), irradiance transmitted through the leaf canopy; \( R_o \), irradiance incident on the leaf canopy; \( \alpha \), leaf angle; \( \beta \), solar angle.

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In this paper, simple calculations are presented to highlight the possible bias in the midday determination of an empirical $k$ value for use in Beer’s Law (Eq. [1]). These calculations were done assuming only direct beam solar radiation to highlight the possible discrepancies in the approaches. Similar computations can be made for diffuse sky radiation conditions once the angular distribution of the incident diffuse radiation has been defined (see Appendix).

Two possible approaches are considered to remove the “midday bias” in the extinction coefficient for use in estimating radiation use efficiency. The first approach is to derive a correction factor that can be applied to midday measurements to better represent daily, integrated radiation interception. The second approach is to estimate the appropriate time of day to make measurements that directly give estimates of daily interception of integrated radiation.

**REVIEW OF THEORY**

**Instantaneous Radiation Interception Model**

Empirical estimations of the radiation extinction coefficient in Eq. [1] continue to be made because the exponential model often fits the radiation interception data very well. Indeed, theoretical analyses have shown that an exponential model is appropriate (deWit, 1965; Duncan et al., 1967), although the assumptions of the derivation are different from that of Beer’s Law. The derivation of Duncan et al. (1967) is straightforward starting with an assumed random distribution of leaf area segments within horizontal layers. From geometrical descriptions of the angle of the incident radiation above the horizon ($\beta$) and the angle of the individual leaf elements above the horizon ($\alpha$), they derived the following equation.

$$\frac{R_o}{R_o^a} = 1 - \exp[-LG\sin(\beta)] \quad [2]$$

where: $G$ = leaf shadow projection calculated from $\beta$ and $\alpha$. Duncan et al. (1967) presented a table of values for $G$ computed for various combinations of $\beta$ and $\alpha$. Comparison of Eq. [1] and [2] shows that the empirical extinction coefficient for a leaf canopy ($k$) should not be expected to be a constant through the day or through the growing season. The value of $k$ is actually a summary term for $G/\sin(\beta)$, and hence, varies with both $\beta$ and $\alpha$.

Equation [2] describes the instantaneous radiation interception for a leaf canopy with a horizontally random distribution of leaf elements. This equation is dependent both on $\beta$ and $\alpha$. Of course, the value of $\beta$ for direct beam radiation depends on the latitude at which the data are collected, time of year, and the time of day. The influence of $\alpha$ is expressed through a complex interaction with $\beta$ in the calculation of $G$, which was presented by Duncan et al. (1967). Their calculations of $G$ were presented in tabular form but to facilitate the comparisons here, regressions of $G$ against $\beta$ were done for assumed $\alpha$ of 30°, 45°, and 60°. Of course, the regressions could be extended to other $\alpha$, but for many crop canopies the average $\alpha$ are in the range of 30° to 60°.

**Weighted Daily $k$ Values**

The instantaneous value in Eq. [2] of $[G/\sin(\beta)]$, which will be referred to as $k_o$, was calculated for midday and compared to estimates of a daily weighted value that could appropriately represent $k$ in Eq. [1]. The values of daily $k$ ($k_d$) were based on integrated incident and intercepted radiation over the daily cycle calculated from instantaneous values obtained from Eq. [2].

The first step in these calculations was to define the incident radiation through the daily cycle. The exponential equation presented by Goudriaan (1982) for a humid sky was used to calculate the incident photosynthetically active radiation (PAR) at each time step based on $\beta$.

$$R_o^a = 640 \sin(\beta) \exp[-0.15/\sin(\beta)] \quad [3]$$

where; $640 = \text{PAR irradiance above the atmosphere (J m}^{-2} \text{s}^{-1})$. The value of $G$ at each time step was obtained based on $\beta$ and $\alpha$ using the regression equations presented in Fig. 1.

The second step was to calculate the amount of transmitted radiation ($R_o$) using Eq. [2] for each 10-min interval throughout the day. The instantaneous values of $R_o^a$ and $R_o$ were summed over the day and used to calculate daily $R_o/R_o^a$. The daily value of $R_o/R_o^a$ was then used to calculate $k_d$ by rearrangement of Eq. [1]:

$$k_d = -\ln(1 - R_o/R_o^a)/L \quad [4]$$

Values of $k_d$ were calculated for $L$ values of 0.5, 1, 3, and 5. In addition, the response of $k_d$ to variations in $\alpha$ (30°, 45°, 60°) and latitudes (20°, 30°, and 40° N) were calculated. These calculations were done for day of year of 180, 220, and 240 d, which are representative of times during the major crop growth period in the northern hemisphere.

**RESULTS AND DISCUSSION**

The value of $G$ changed smoothly with changes in $\beta$ (Fig. 1). The least variation in $G$ with changing $\beta$ occurred with the greatest $\alpha$. The results of Duncan et al. (1967) were readily represented for each $\alpha$ by fourth-order polynomials, which had $R^2$ values of 0.96 or greater.

The variation in $k$ through the daily cycle was calculated for the three assumed $\alpha$ (Fig. 2). These results
showed that during the midday period at 30° latitude, Day 180 and α equal to 30°, the value of $k_i$ was stable for an extended period. These results were obtained because the decreases in $\sin(\beta)$ that occurred on either side of solar noon were approximately matched by decreases in the value of $G$. These results indicated that determination of $k_i$ based on midday measurements of canopy radiation interception gives a stable result and there is likely not a need to consider the exact time at which the measurements are made at midday.

Since the value of $k_i$ was found to be stable under a range of fairly high sun angles, stability in midday $k_i$ was also found when changing latitude from 30° to 20° or to 40° (Fig. 2). Similarly, midday $k_i$ on Day 210 or 240 was very similar to those calculated on Day 180. These results indicated that midday $k_i$ is likely to be nearly constant through the growing season if leaf angle does not change.

However, stability in $k_i$ does not extend over the whole day. At the ends of the day with lower β, $G$ does not continue to decrease (Fig. 1) so the values of $k_i$ increase with further decreases in β. Due to the fact that the changes in $G$ with β are different for each leaf angle, the time at which $k_i$ increases is different among the different α. Stability in $k_i$ only exists for about 4 h around solar noon for α of 60°, 6 h for α of 45°, and 8 h for α of 30°. Hence, the time restrictions on measuring light interception in canopies to obtain a "representative" midday value for $k_i$ are somewhat more limited as the values of α increase.

The midday values of $k_i$ decrease as α increases (Fig. 2). That is, more erect leaves allow greater radiation penetration deeper in the canopy than more horizontal leaves. However, this situation reverses at the end of the day in that $k_i$ increases at the ends of the day resulting in greater $k_i$ for the erect leaf crop than for the horizontal leaf crop. The time of crossover in $k_i$ among canopies of different α in the example in Fig. 2 at latitude 30° and Day 180 was at about 4.5 h on either side of solar noon. At the ends of the day, therefore, the deeper penetration of radiation occurs in canopies with more horizontal leaves.

The values of $k_i$ increase substantially at the ends of the day due to low β. That is, the canopy readily intercepts the radiation by top leaves when the incident angle is low. As a result, virtually all radiation incident to the canopy at the ends of the day is intercepted by the leaves (Fig. 3).

Since $k_i$ increases so greatly at the ends of the day, the midday values of $k_i$ fail to account for the much higher radiation interception at low β. Consequently, $k_i$ values estimated from midday measurements of radiation interception by the crop canopy are an underestimate of intercepted solar radiation on a daily basis. This, in turn, results in an overestimation of radiation use efficiency when calculated based on midday $k_i$.

Equation [4] was used to calculate $k_d$ based on the amount of total radiation intercepted in the day relative to incident radiation. Calculation of $k_d$, however, is not independent of $L$ because the relative amount of radiation intercepted at various times through the day depends on $L$. That is, increased $L$ results in proportionally more radiation being intercepted at midday than at the ends of the day so the estimate of $k_d$ is decreased. This result is indicated in Fig. 4 where the value of $k_d$ decreases to some extent as $L$ increases. The largest change in $k_d$ with $L$ occurred with the greatest α. There was only small change in $k_d$ with changes in latitude and day of the year during the growing season.

Values of $k_d$ and midday $k_i$ were calculated for a range of latitudes and day numbers in addition to α and $L$. Of
course, in all cases, the value of $k_d$ was greater than midday $k_i$ (Fig. 5). That is, the interception of radiation was greater over the entire daily cycle than that estimated from midday $k_i$. Based on these calculations for clear sky conditions, values of $k_d$ were linearly correlated ($R^2 = 0.98$) with midday $k_i$. The deviation between the two variables increased at lower values, however. These results are fully consistent with the experimental observation of Muchow (1985) that the deviation between a daily estimate of radiation interception and midday estimates was greater at lower levels of fraction intercepted radiation.

Based on the regression between $k_d$ and midday $k_i$ (Fig. 5), $k_d$ for the case of $\alpha$ equal to 60° was calculated to be about 20% greater than midday $k_i$. However, there was considerable variation in the $k_d$ value at $\alpha$ equal to 60° resulting from the sensitivity of $k_d$ to $L$. This variation in $k_d$ at low values of midday $k_i$ indicates that a correction simply based on the regression in Fig. 5 may not be adequate. This concern would be greatest at low $L$ when calculation of radiation interception is especially sensitive to the value of the extinction coefficient (Eq. [1]). To fully make the correction in midday $k_i$ to estimate $k_d$, it would likely be necessary to account for the influence of $L$ on the daily extinction coefficient.

An alternative to correcting midday $k_i$ to obtain $k_d$ is to make measurements of radiation interception by the leaf canopy at times in the day when $k_i$ approximates the value of $k_d$. Clearly, in this case the measurements would need to be made near the ends of the day when $k_i$ is greater than midday $k_i$. The output $k_i$ for each 10 min though the day was scanned to find the time at which the $k_i$ matched the value of $k_d$ for each set of conditions. The appropriate time to determine $k_i$ as a direct estimate of $k_d$ are given in Fig. 6 for the morning period only because it is more likely in most climates that the sky will be more clear in the morning than in the afternoon. As already discussed, the change in $k_i$ through the daily cycle is dependent on $\alpha$. Increases in $k_i$ occur closer to midday in an erect leaf canopy than in a more horizontal

leaf canopy. Hence, the time of day to measure radiation interception by a canopy to derive an estimate of $k_d$ is dependent on leaf angle. The calculated time to measure radiation interception to estimate $k_d$ (Fig. 6) for crops with various leaf angles was approximately 0735 to 0755 h for a 30° crop, 0815 to 0835 h for a 45° crop, and 0845 to 0905 h for a 60° crop.

The results of this analysis are discouraging in regards to using the approach of measuring radiation interception under clear sky at alternate times of the day to estimate directly $k_d$. The difficulty in this alternate experimental approach is that $k_i$ changes rapidly at the ends of the day (Fig. 2). Therefore, the time window for making the “$k_d$ measurement” is narrow and only a limited number of measurements are possible within the short time frame. Further, “$k_d$ measurement” is sensitive both to $\alpha$ and $L$, so that a substantial amount of information is needed about the leaf canopy to define the appropriate timeframe to make measurements. Therefore, it seems likely that the better approach to estimating $k_d$ would be to determine midday $k_i$ and make the appropriate correction, including an accounting for $L$ if such data are available.
APPENDIX: DIFFUSE RADIATION

Calculation of an extinction radiation for diffuse radiation is complicated because of the range in possible distributions of radiation originating from various sky segments. A number of complicating atmospheric conditions as well as the angle of the sun gives a wide number of possibilities for the angular distribution of incident diffuse radiation.

The simplest assumption for diffuse radiation is to assume a fully overcast condition with radiation originating uniformly from all segments of the sky. Therefore, the diffuse radiation can be assumed to originate from a hemisphere and the calculation of $k$ is independent of time of day, time of year, or latitude. The fraction of radiation ($F$) received from each sky segment between angles $\alpha_1$ and $\alpha_2$ can be calculated using the following equation.

$$F = \sin(\alpha_2)^2 - \sin(\alpha_1)^2 \quad [A1]$$

The extinction coefficient considering the entire sky was obtained by calculating the intercepted radiation originating from each $10^\circ$ segment of the sky. Summing the intercepted radiation from each sky segment gives total intercepted radiation, which was then used in Eq. [3] to obtain a $k_d$ for diffuse radiation.

Figure 7 presents the estimates of $k_d$ under a uniformly diffuse sky for various $\alpha$ and $L$. This figure is similar to Fig. 4 in that both increasing $\alpha$ and increasing $L$ resulted in decreased $k_d$. The influence of $L$ on $k_d$ is greater under diffuse radiation than direct radiation because of the proportion of total radiation coming from comparatively low angles with diffuse radiation. Therefore, at low $L$ much of the intercepted radiation is from low sky angles resulting in a large value for $k_d$. Increasing $L$ increases the interception of the radiation originating from higher sky angles and causes $k_d$ to decrease. For the assumed uniform source of diffuse radiation, these calculations demonstrate that $L$ of the vegetation, especially with higher leaf angles, may have substantial impact on the value of $k_d$. Therefore, it is not possible to assume a constant $k_d$ even under diffuse radiation when there are changes in $\alpha$ and/or $L$ through the growing season.

REFERENCES